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Minimum Energy Requirements for Space Travel[1]

By

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(With 12 Figures)

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Abstract — Zusammenfassung — Résumé

Minimum Energy Requirements for Space Travel. The minimum energy requirement for many space missions is calculated and expressed as velocity requirement of a rocket vehicle, which is supposed to fulfill them. This gives a fast possibility for a preliminary outline of an optimum vehicle, or for an approximation of the payload capability of a given vehicle.

Of course, mission flight times can be reduced by utilizing more than minimum energy. This is particularly pronounced in lunar and interplanetary transfers.

Minimalenergie-Erfordernisse beim Raumflug. Die Minimalenergie-Erfordernisse für viele Raumflugprojekte werden berechnet und als Geschwindigkeitserfordernisse eines Raketenfahrzeuges ausgedrückt, das diese Bedingungen erfüllen soll. Dies bietet die rasche Möglichkeit für einen vorläufigen Entwurf eines optimalen Fahrzeuges oder für die angenäherte Berechnung der Nutzlast-Aufnahmefähigkeit eines gegebenen Fahrzeuges.

Selbstverständlich können die einem Projekt zugeordneten Flugzeiten verringert werden, wenn mehr als die Minimalenergie verwendet wird. Dies wird besonders deutlich bei Übergangsbahnen zum Mond oder zwischen Planeten.

Spécifications minimum d'énergie pour voyages interplanétaires. Les spécifications minimum d'énergie sont converties en spécifications de vitesse de la fusée destinée à remplir une mission. Ceci permet une estimation rapide de la charge payante et de la configuration optimale.

Il est évident que les temps de vol peuvent être réduits par un excédent d'énergie, particulièrement pour les vols lunaires et les transferts interplanétaires.

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[1] Statements and opinions are to be understood as individual expressions of the author and do not necessarily reflect the views and opinions of ABMA.

This study is concerned with some of the physical fundamentals of space flight. It is in no way related to any project now being worked on by the Army, nor should it be construed as a description of any future project which may be assigned to the Army.

There is no reference made to any specific hardware now in the making.

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I. Introduction

For an idealized step rocket moving along a straight line free of exterior forces holds the well-known equation

$$V_{id} = \sum_{i=1}^{n} c_i \ln r \, .$$

This velocity shall be called "ideal velocity capability of the vehicle."

A real rocket moving near the Earth's surface does not reach V_{id} as there are velocity losses due to drag and the gravitational field. It is easy to show that despite the increase in altitude, which gives a potential energy gain, there is still an energy loss due to gravity. The magnitude of losses depends upon the geometry of the ascent path and on the flight program. In space-flight missions involving high acceleration propulsion systems (order of one g)—and only such shall be of interest here—the usual flight method is to impart to the vehicle a certain mission dependent energy E_1 (say, escape energy), make a correction on the way to the target, and apply the terminal maneuver at the target (perhaps brake the energy E_2 of impact on the moon). The energy E_1 corresponds now to a velocity V_1 and if we add some empirical or semi-empirical corrective figures for drag and gravity loss, usually taking account of the variation of specific impulse with ambient pressure by taking some convenient mean value for jet velocity in the first stage, we can quote an ideal velocity figure which is necessary for the vehicle to have in order to perform the escape: "ideal velocity requirement of the mission." Adding to this value for escape some empirical number for correction, and a figure representing E_2, we get the ideal velocity requirement for a soft lunar landing.

If a vehicle shall do a certain mission, then, of course, the ideal velocity capability of the vehicle must be larger, or equal to, the ideal velocity requirement of this mission.

In those requirements a correction is necessary since we can get from Earth rotation up to 460 m/sec free; this means that the "ideal requirement" can be 460 m/sec less than one would think from the pure energetic point of view. But often because of flight geometry and other considerations only 300 m/sec can be utilized.

In order to have a yard stick for the vehicle capabilities, where are we now? As known, lunar space probes have been successfully fired, meaning that $V_{id} = 12.45$ km/sec is available. As there is much talk about a Venusian space probe, $V_{id} = 13.0$ km/sec can be expected. What is the ultimate limit of chemical rocket vehicles? Assuming a launch weight of $10 \cdot 10^6$ lb, and a gross payload (including guidance, etc.) of 1000 lb, we have a growth factor of 10^4. For a four-stage vehicle this means a payload ratio per step of 0.1. Assuming a structure ratio of 0.04, we have for one stage a mass ratio of about 7.14, leading with a jet velocity of 4.5 km/sec to $V_{id} = 35.4$ km/sec. Applying such tricks of the trade as orbital technique, the figure may rise to 45 km/sec, with some small increases possible due to the use of orbital technique at the target. As perturbation maneuvers can reduce the requirements slightly, we can understand them to be a small increase in velocity capability. The "feasibility limit", then, can somewhat arbitrarily be set perhaps at $V_{id} = 50$ km/sec.

As the problems of such a vehicle are stupendous (as all of you realize), I feel, therefore, that a practical limit will be lower. Fortunately, we do not need such high ideal velocities within the solar system, as the "principle of mission staging" is a way to avoid too high requirements. For example, you go to the lunar surface using many landing vehicles, and assemble there an Earth-return vehicle out of the useful payload of your landers. So you can return to Earth, using several vehicles of one-way capability instead of one vehicle of two-way capability. Therefore, as an estimate, the practical limit for a chemically-propelled rocket vehicle may be near $V_{id} = 25$ km/sec.

Using a "conventional" nuclear propulsion system as it is seen today (nuclear reactor heating a working fluid, which expands through a nozzle), we could perhaps get 80 km/sec instead of the 50 for the chemical system. The more practical limit could conceivably be near 30 km/sec.

The low-acceleration systems could have still higher velocity capabilities, but part of this is used up against the higher gravity losses. Perhaps as ball-park figures, 100 km/sec as ultimate limit, and 50 km/sec as practical limit, can be envisioned.

To meet still higher requirements, very exotic propulsion systems must be used; for example, Prof. SÄNGER's photon propulsion.

II. Velocity Requirements for Earth-Bound Missions

For comparision only, some approximate figures have been computed, which give range versus ideal velocity capability for ballistic-type missiles (Table I).

III. Velocity Requirements for Satellite Missions (Circular Orbits)

A. Nonrotating Earth (Polar Orbits)

Assuming a HOHMANN transfer (Fig. 1), a first kick is necessary to throw the payload up to orbital altitude:

$$\delta_1' = \left(\frac{\gamma M}{R}\right)^{\frac{1}{2}} \left(\frac{2r}{r+R}\right)^{\frac{1}{2}}.$$

Introducing $\dfrac{\delta'}{\left(\frac{\gamma M}{R}\right)^{\frac{1}{2}}} = \delta$, and $\dfrac{r-R}{R} = k$ comes

$$\delta_1 = \left(\frac{2+2k}{2+k}\right)^{\frac{1}{2}}. \qquad (1)$$

The arrival velocity at apogee is

$$V_a = \left\{\frac{2}{(2+k)\;(1+k)}\right\}^{\frac{1}{2}}. \qquad (2)$$

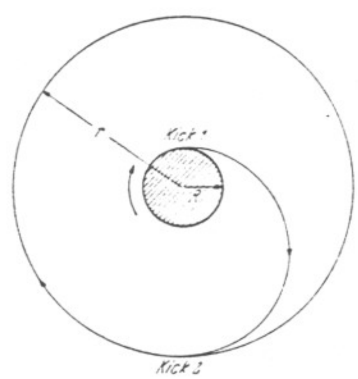

Fig. 1. HOHMANN transfer. Circular velocity for r = R equals $[(\gamma M)/R]^{\frac{1}{2}} =$ 7910 m/sec for Earth

So a second kick to circularize must be given

$$\delta_2 = \frac{1}{(1+k)^{\frac{1}{2}}} \left\{1 - \left(\frac{2}{2+k}\right)^{\frac{1}{2}}\right\}. \qquad (3)$$

The total ideal requirement for the mission then is

$$\Delta_1 = \delta_1 + \delta_2 = \frac{k + (1+k/2)^{\frac{1}{2}}}{(1+k)^{\frac{1}{2}}\;(1+k/2)^{\frac{1}{2}}} \approx 1 + k/2, \text{ for small } k. \qquad (4)$$

Introducing for the moment $(1+k/2)^{\frac{1}{2}} = x$, this can be written

$$\Delta_1 = \frac{2x^2 - 2 + x}{(2x^2-1)^{\frac{1}{2}}\, x} = \frac{2x - 2/x + 1}{(2x^2-1)^{\frac{1}{2}}}\,; \quad \text{from} \quad \frac{\partial \Delta_1}{\partial x} = 0$$

$$\rightarrow x^3 = 3x^2 - 1\,.$$

The solution is $x \approx 2.88$, leading to $k = 14.58176$ for the most difficult circular orbit with $\Delta_1 \approx 1.5362$.

Using a three-kick transfer (Fig. 2), the first kick is used to throw the payload to infinity.

$$\delta_1 = \sqrt{2}\,. \tag{5}$$

The second kick at infinity is a zero-adjustment kick; the third kick brakes the arrival speed (escape) to circular speed:

$$\delta_3 = (\sqrt{2} - 1)\,\frac{1}{(1+k)^{\frac{1}{2}}}\,. \tag{6}$$

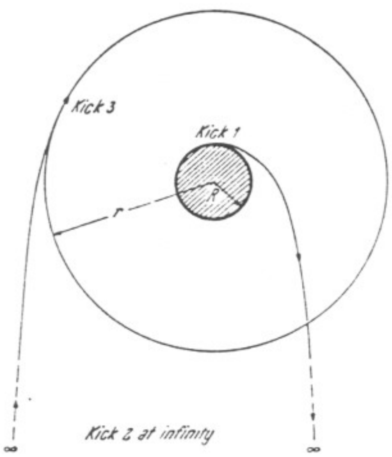

Fig. 2. Three-kick transfer

In total, we have used

$$\Delta_2 = \sqrt{2} + \frac{\sqrt{2}-1}{(1+k)^{\frac{1}{2}}}\,. \tag{7}$$

Comparing Δ_2 and Δ_1, there is $\Delta_2 < \Delta_1$ for $k > 10.94$, which means, that the three-kick transfer is superior for very high orbits.

So, energy-wise, the circular orbit at $k = 10.95$ is the most difficult one, with $\Delta = 1.5340$, if the three-kick transfer is used for larger altitudes. (These considerations are of theoretical value only, since $k = 10.94$ corresponds to satellite altitudes of no interest, and besides the three-kick transfer has other serious disadvantages: Long travel time, and extreme sensitive to the second kick.)

B. Rotating Earth

If the firing occurs from a latitude λ under an azimuth a (East of North) then there is an assistance given from Earth rotation of approximately

$$V_E = 450 \cdot \cos\lambda \sin a \;[\text{m/sec}]\,. \tag{8}$$

C. Change of Orbital Plane

Only a simple case shall be considered here; viz, to go into an equatorial orbit by launching from the latitude φ under an azimuth of $a = 90°$. Ascent is via HOHMANN transfer to a waiting orbit at 200 km altitude. This is necessary in order to place the apogee of the transfer ellipse from the waiting to the final orbit over the equator.

$$\Delta_1 = 1.0157 \approx 1 + \frac{k}{2}\,.$$

An escape from there would require $\Delta_{Esc} = (\sqrt{2} - 1)\left(1 - \frac{k}{2}\right)$.

So the total escape $\Delta = \sqrt{2} + (2 - \sqrt{2})\,\frac{k}{2}$.

The direct escape is $\sqrt{2}$—so there is a waiting-orbit loss for escape of

$$\Delta_L = (2 - \sqrt{2})\,\frac{k}{2} \approx 0.0092\,.$$

Because of maneuvering, I will take $\Delta_L = 0.01$ as "typical waiting-orbit loss."

The upper kick for circularization is given by

$$\delta = \frac{\sqrt{1 + \frac{k}{2}} - 1}{\sqrt{1 + \frac{k}{2}}\,\sqrt{1 + k}}\,. \qquad (9)$$

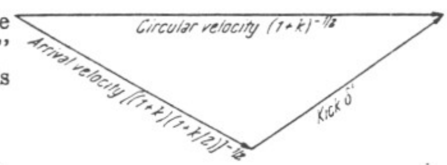

Fig. 3. Change of inclination

If there is a plane change φ involved (see Fig. 3), then the kick to both circularize and accomplish the plane change is

$$\delta^1 = \sqrt{\delta^2 + \frac{4\sin^2\varphi/2}{(1+k)\sqrt{1+k/2}}}\,. \qquad (10)$$

The total velocity required on a rotating Earth then is

$$\Delta_{1\varphi} = \Delta_1 - \frac{450}{\left(\frac{\gamma M}{R}\right)^{1/2}} + (\delta^1 - \delta) + \frac{900}{\left(\frac{\gamma M}{R}\right)^{1/2}}\sin^2\frac{\varphi}{2} + 0.01\,S(\varphi) \qquad (11)$$

where $S(\varphi) = 0$ for $\varphi = 0$ and
$S(\varphi) = 1$ for $\varphi \neq 0$
[δ', δ are given by eqs. (10, 11); Δ_1 by eq. (4)].

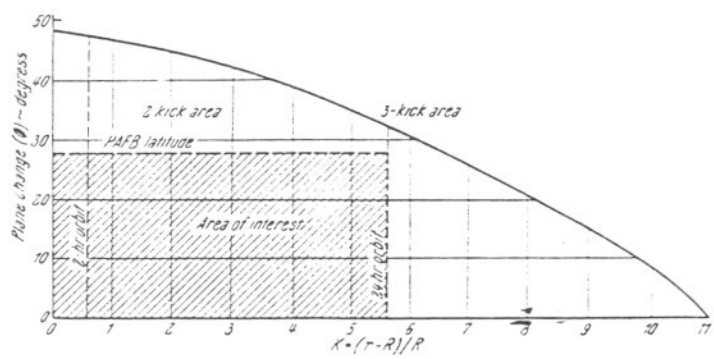

Fig. 4. Three-kick transfer versus two-kick transfer (energy considerations only)

The corresponding expression for the three-kick transfer is

$$\Delta_{2\varphi} = \Delta_2 - \frac{450}{\left(\frac{\gamma M}{R}\right)^{1/2}} + \frac{900}{\left(\frac{\gamma M}{R}\right)^{1/2}}\sin^2\frac{\varphi}{2} \qquad (12)$$

[Δ_2 is given by eq. (7)].

Energywise, we consider the function

$$A(\varphi, k) = \Delta_{2\varphi} - \Delta_{1\varphi}. \tag{13}$$

If $A > 0$, then the HOHMANN transfer is preferable;
$A < 0$, then the three-kick transfer is preferable.

It is easy to show:

1. $k < 10.94, \varphi = 0 : A > 0$
2. $k = \infty$, φ arbitrary: $A = 0$
3. $k =$ arbitrary, $\varphi > 48.3°: A < 0$

For $A(\varphi, k) = 0$, (see Fig. 4).

It is seen that the three-kick transfer appears not to have a field of practical application.

D. Examples

Circular 568 km-orbit, 96-min, equatorial: .... $(\gamma M/R)^{1/2} =$ 7,920 m/sec
Equator-launched, HOHMANN .... $V_{id} = 0.99 =$ 7,841 m/sec
Equator-launched, three-kick .... 1.76 = 13,939 m/sec
$\varphi = 28°$-launched, HOHMANN .... 1.44 = 11,405 m/sec
$\varphi = 28°$-launched, three-kick .... 1,766 = 13,987 m/sec

Circular 24-hr orbit, equatorial:
Equator-launched, HOHMANN .... 1.45 = 11,484 m/sec
Equator-launched, three-kick .... 1.52 = 12,038 m/sec
$\varphi = 28°$-launched, HOHMANN .... 1.51 = 11,959 m/sec
$\varphi = 28°$-launched, three-kick .... 1.526 = 12,086 m/sec
$\varphi = 28°$-launched, 28° inclined orbit .... 55 m/sec to above equator-launched velocities

Escape: equatorial-launched, Eastward .... 10,736 m/sec
polar-launched .... 11.186 m/sec

What does this mean in payload? Let us assume the following arbitrary vehicles for the first mission:

	Type		
	A	B	C
Payload (Container) Weight	100	80	70
Guidance, Control, Instruments	4	11	14
Fuselage	4	11	14
Motor, etc.	4	10	14
Total	112	112	112

We will look at the HOHMANN-transfer missions only
28°-launched, 28° inclined orbit: with $I_{sp} = 300$ sec,

$$55 = 300\, g. \quad \ln \frac{M}{m} \to \frac{M}{m} = 1.0186, \text{ which leads to}$$

$$m = \frac{112}{1.0186} = 109.95, \text{ or fuel used: } 2.05$$

Additional Tankage: 0.2

Payload loss: 2.25 of 100 for Vehicle A.

With an I_{sp} of 400 sec, the payload loss was 1.65 of 100.
In this manner the following table has been computed.

Payloads Including Containers
(HOHMANN transfers only)

	I_{sp} = 300 sec			I_{sp} = 400 sec		
	A	B	C	A	B	C
Equator-launched Equatorial 96-min Orbit	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
28°-Launched, 28° Inclined	97.8%	97.0%	96.9%	98.4%	97.9%	97.6%
28°-Launched, Equatorial	13.4%	—	—	26.4%	8.0%	—
Equator-launched, Equatorial 24-hr Orbit	12.4%	—	—	26.0%	7.5%	—
28°-Launched, 28° Inclined 24-hr Orbit	11.9%	—	—	24.9%	6.1%	—
28°-Launched, Equatorial 24-hr Orbit	7.2%	—	—	19.9%	0.0%	—
Escape, Equator-launched, Due East	22.8%	3.5%	—	35.6%	19.5%	8.0%
Escape 28°-launched, Due East	22.0%	2.5%	—	34.9%	18.6%	7.0%
Escape Polar	16.2%	—	—	29.2%	11.5%	0.0%

This table illustrated several well-known facts:

1. If azimuth of firing is 90°, then only small penalties are involved in going up to 30° off the equator.

2. If higher energy missions are flown, then it is not optimum just to exchange propellant for payload in the last stage. But if this is done, then the result is very dependent upon specific impulse.

3. Even for vehicles of similar performance at one mission, the performance at other missions may vary widely.

E. Recovery of Satellites

Here one kick is assumed to brake the orbital velocity so far that a transfer ellipse is entered the pericenter of which is sufficiently deep in the atmosphere, so that further braking is done by atmospheric drag. Final descent could employ lift, or a parachute, or in the case of some types of instruments simply impact.

F. Conversion of "Ideal Required Velocity" to "Required Velocity"

Certain correction figures have to be added, which are of an empirical nature. Those correction figures take care of:

1. Gravity loss and drag loss: 1500 m/sec seem to be a usable figure for escape missions and large vehicles. (Of course, this number depends on vehicle shape, trajectory and acceleration program.)

2. Maneuvering Reserve: This has to be estimated for every mission.

3. Unusable propellant residuals and mixture ratio shifts: With control of mixture ratio and trapped propellants, 3% of the ideal velocity should suffice.

IV. Velocity Requirements for Space Probes

By space probe a vehicle is meant which is used for space research without necessarily approaching a planet or any other celestial body. Therefore, the guidance and space navigation problems are greatly simplified. To go from Earth to the Moon, escape velocity $\left(2\,\frac{\gamma M}{R}\right)^{1/2}$ is approximately necessary, the minimum for direct transfers being about 100 m/sec lower. If we look for the circular orbit of the same energy requirement, we have to solve $\frac{k+\sqrt{1+k/2}}{\sqrt{(1+k)\,(1+k/2)}}=\sqrt{2}$, from which comes $k \approx 2.303$.

So we can conclude, that, for a simple Earth-escape experiment, about the same payload can be carried as into the circular orbit of $k=2.303$. In practice even more can be carried, as the guidance system should be simpler and, therefore, lighter.

For interplanetary probes, obviously, more energy is required than for probes in near-Earth Space. It is easy to show, that, disregarding Earth, a minimum perihelion velocity of 32.83 km/sec at Earth distance (149.10^6 km) from the Sun is necessary in order to place the aphelion out to 230.10^6 km (Mars distance). This is 32.83—29.8 = 3.03 km/sec above local circular velocity (Earth velocity, going around Sun). Therefore, the minimum required launch velocity is

$$\sqrt{11.1862^2+3.03^2}=11.5893 \text{ km/sec}$$

which is only 403.1 m/sec more than the simple Earth escape probe.

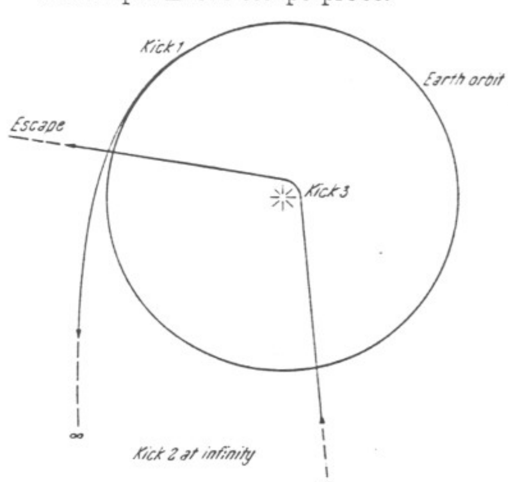

Fig. 5. Solar system escape – Hi-performance vehicle

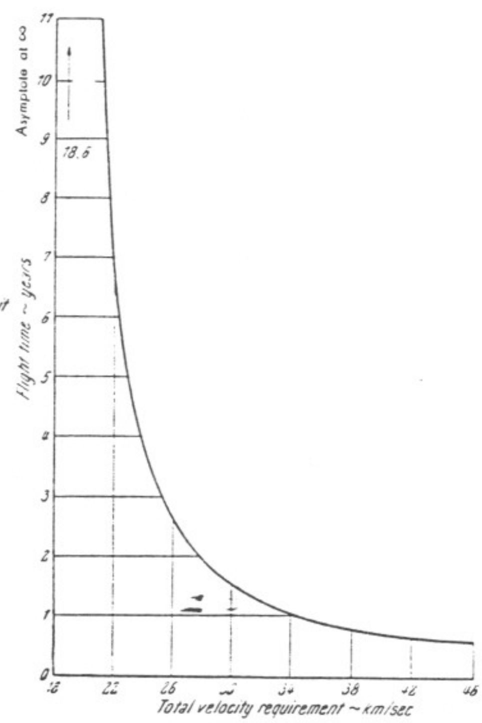

Fig. 6. Solar probe: Flight time versus total velocity

For Venusian probe, put 2.4 km/sec instead of the 3.03 km/sec for Mars, resulting in $V_{min} = 11.4408$ km/sec, only 254.6 km/sec above escape velocity.

The minimum requirement for a Mercury probe is 13.504 km/sec or 2.318 km/sec excess over Earth escape.

The minimum to escape the solar system is to have a residual velocity of $(\sqrt{2}-1)\cdot 29.8 = 12.3436$ km/sec. This leads to a total minimum requirement of 16.6582 km/sec which is 5.472 km/sec over simple Earth escape.

Assuming we had a vehicle of a capability of $V_{id} = 50$ (100) km/sec. To leave the solar system (Fig. 5) it would be best to apply a first impulse in order just to

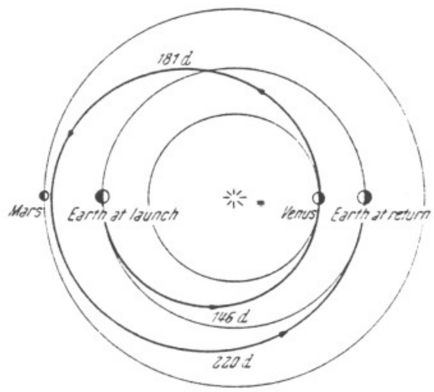

Fig. 7. Hohmann's round-trip trajectory

Maneuvers:

1. Brake original 29.8 km/sec of Earth by	2.4 km/sec
2. Increase on passing Venus to 39.4 km/sec by	1.8 km/sec
3. Increase on passing Mars to 24.8 km/sec by	2.3 km/sec
4. Brake 31.5 km/sec arrival at Earth by	1.8 km/sec
Total velocity requirement	17.6 km/sec
Total flight time	547 days

leave the system: 16.66 + losses ≈ 20 km/sec. At infinity, zero kicks would adjust for a return to just graze the Sun. Arrival velocity would be local escape velocity = = 617.5 km/sec at 700,000 km from the Sun's center. To this we add the remaining 30 (70) km/sec, leading to a velocity remaining at infinity of 195 (302) km/sec. Travel time to the nearest fixed star, about 4 light-years distant, would now be about 6150 (3980) years, or total mission time about 6200 (4000) years. Therefore, with the highly advanced vehicles which were assumed, not even the nearest fixed stars are within reach. (In order to get to them in 40 years of transit time, the ideal velocity capability of the vehicle must be of the order of 30,000 km/sec.)

In solar probes we have to differentiate between three classes:

1. Probes in near solar space. A good example of this is the Mercury probe, for which we found V_{id} min = 13.504 km/sec. The flight time is about 111 days.

2. Direct probe to the Sun: The Earth orbital velocity is braked, and the vehicle drops in vertically to the Sun. The required ideal velocity is high— 31.8 km/sec with a flight time of only 65 days.

3. Indirect solar probe. Here a solar system escape must be performed first and then a drop towards the Sun. The ideal energy required is only 16.66 km/sec, but the flight time is infinite. For practical cases, you would, of course, not go quite to infinity, resulting in a higher velocity and lower flight time required (see Fig. 6).

More sophisticated types of probes are those performing round trips, two of which may be of special interest:

Hohmann Roundtrip (Fig. 7).
Crocco Roundtrip (Fig. 8).
As these probes can conceivably return to Earth, a manned version of such an expedition may be interesting.

V. Lunar Flights

Six types of flights must be considered:
1. Probes
2. Hard impacts
3. Circumlunar Flights
4. Lunar Satellites
5. Soft Landers
6. Earth-Moon Return Flights.

Probes and hard impacts differ mainly in guidance accuracy required. Therefore, no more additional information seems necessary here.

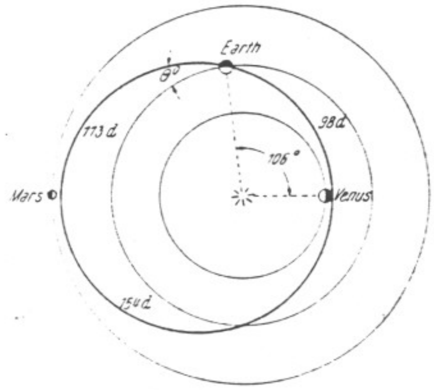

Fig. 8. Crocco's round-trip trajectory

Total velocity requirement	15.968 km/sec
Total flight time	12.5 to 13.5 months
Possible launch time	June 1971
ϑ	16.04°

For the circumlunar flight the energy requirement, apart from maneuvering, is similar to Earth escape. If a return to Earth is planned, then, because of the critical atmospheric re-entry, ample maneuvering fuel should be provided for.

Let us look for a moment at some characteristic data for transfer trajectories in the Earth-Moon system:

Injection angle: near horizontal

	Cutoff Velocity in Inertial Earth-Centered Space at 200 km Altitude	Flight Time	Unbraked Lunar Impact Velocity
1	10,881 m/sec	~10 years	2,325 m/sec
2	10,920 m/sec	~ 5 days	2,500 m/sec
3	10,970 m/sec	$2^1/_2$ days	2,705 m/sec
4	11,015 m/sec	51 hrs	2,886 m/sec
5	11,100 m/sec	41 hrs	3,179 m/sec

Remarks:

1. Absolute minimum injection velocity to reach the Moon, from JACOBI's Integral.
2. Minimum for direct Earth—Moon transfer.
3. Two-and-a-half day transfer.
4. Injection velocity equal local escape velocity.
5. "Fast" trajectory.

Some Data:

Mean Earth Radius	6,371.1 km
Mean Lunar Radius	1,738 km
Escape velocity, Earth, zero altitude	11,186 km/sec
Escape velocity, Moon	2,374 km/sec
"Mean" Earth—Moon distance	384,412.3 km

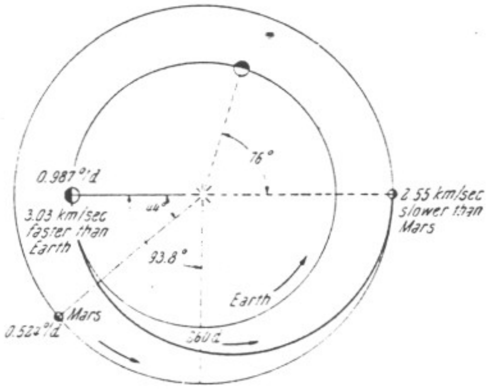

Fig. 9. Earth—Mars mission profile (HOHMANN ellipse)
Opposition occurs (44)/(0.987—0.524) = 95 days = 93.8° after launch. The waiting time is in this case (360—2.76)/(0.987—0.524) = 449 days. So a return mission last (260/449/260) = 970 days

From the guidance point of view, the "slow" trajectories show large deviations for small injection errors. Therefore, trajectory No. 4 seems to be a good compromise, with 3 being a competitor.

Upon arrival for a lunar satellite, we have to brake:

Satellite Altitude	Trajectory	Circular Velocity	To be Braked
$\frac{h}{R_{Moon}} = 0$	$2^1/_2$ days	1678 m/sec	1027 m/sec
0	51 hrs	1678 m/sec	1208 m/sec
0.25	$2^1/_2$ days	1502 m/sec	985 m/sec
0.25	51 hrs	1502 m/sec	1181 m/sec
1	$2^1/_2$ days	1188 m/sec	932 m/sec
1	51 hrs	1188 m/sec	1160 m/sec

A soft lunar landing vehicle has to brake the total impact velocity by rocket action.

The return flight is, energy-wise, much simpler; because, at Earth's side probably the atmosphere can be used for re-entry braking. However, corrective fuel should be provided as the ideal re-entry conditions must be met rather closely.

VI. Orbital Technique

It is necessary to say a few words on the use of orbital technique.

a) Upon Departure

For example, in the 96 min-orbit a space vehicle could be assembled. This, then, could take advantage of the energy it already has, and a noticeable structural advantage should result from the possibility of using relatively low accelerations, and from the absence of aerodynamic considerations. Maneuvers into the 96-min

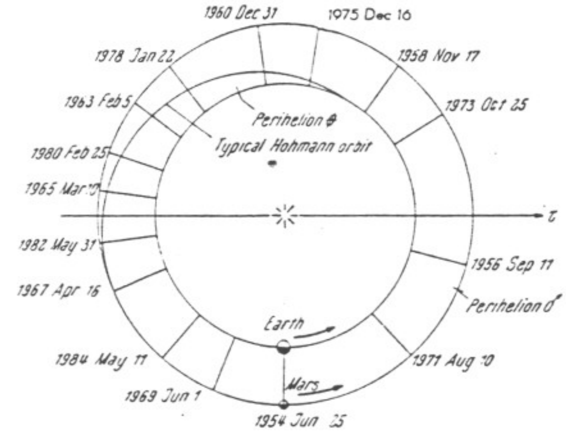

Fig. 10. Opposition of Mars and HOHMANN ellipse injection dates

HOHMANN *ellipse injection dates:*

23 Aug 1958	23 Feb 1969	7 Oct 1977
25 Sep 1960	21 May 1971	29 Oct 1979
16 Oct 1962	7 Aug 1973	4 Dec 1981
9 Nov 1964	12 Sep 1975	28 Jan 1984
25 Dec 1966		

Notes: Maximum error in injection dates: 1958—1971, 3 days; 1971—1984, 5 days. The opposition dates were obtained by extending the ephemeris of the Earth and Mars by a numerical integration method. This ephemeris was extended through 1984 by use of a graph showing synodic period of Mars versus longitude of opposition, prepared from the American Ephemeris and Nautical Almanac. The injection dates were obtained by use of a graph showing opposition time minus injection time versus longitude of opposition. This graph was prepared by choosing a multiple of conjunction times of the HOHMANN ellipse and the orbit of Mars and from this, calculating the period and the theoretical opposition times and longitudes. These were then plotted to obtain the graph

orbit and the resulting high cutoff altitude for the powered phase leaving the orbit will result in some small energy losses.

b) Upon Arrival

At the target site, going first to an orbit and from there down to the surface has potential advantages:

1. Perhaps better control of landing area.
2. Higher safety of mission success, even if the landing fails.
3. Saving of energy, if return fuel is left in orbit and picked up during the return phase.

Disadvantages are:

1. More complicated over-all scheme.
2. All braking could be done aerodynamically if there is an atmosphere; if the braking to orbit is done by rocket, usually much more unfavorable conditions exist.

c) *Upon Return*

Upon return to Earth, the following possibilities exist:

1. Direct re-entry to the atmosphere.

2. Return to 96-min orbit, thereby using rocket braking. The disadvantage of using fuel is partly compensated for by the advantage of using a special vehicle for transferring and receiving.

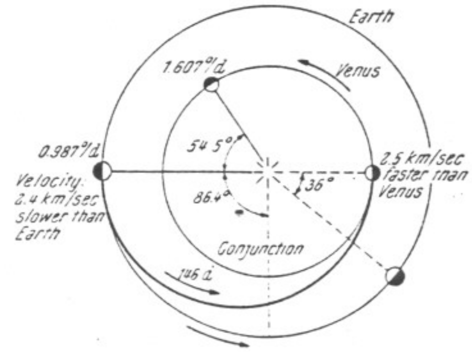

Fig. 11. Earth—Venus mission profile (HOHMANN ellipse)

Inferior conjunction is (54.5)/(1.607—0.987) = 87.6 days after launch; the angle is 87.6·0.987 = 86.4°. From this follows the rule: Launch 86.4° = 87.6 days before inferior conjunction.
For the return flight, we need to have Earth leading by 36°. (In above picture, we simply reverse all directions for the return flight.) So Venus has to catch up 360—2.36 = 288°, which takes (288)/(1.607—0.987) ≈ 460 days. So a return mission lasts 146/480/146 = 750 days

3. Return to another orbit, being picked up there and brought to the 96-min orbit, from where transportation to the Earth's surface is provided. A very interesting return orbit of this type is the following elliptic orbit, which often needs only little rocket braking:

Elliptic Orbit Properties

Perigee: 568 km altitude (circular velocity: 7581 m/sec, escape velocity: 10,721 m/sec).
Velocity: 10,431 m/sec (2850 m/sec above circular velocity).
Apogee: 120,143.4 km from Earth's center.
Eccentricity: 0.89321044.
Major axis: 65,044.7 km.
Minor axis: 29,247.0 km.
Period: 45.848 hr.

So all the braking necessary is 10,721—10,431 = 290 m/sec. Of course, now the pick-up vehicle has to have an ideal minimum capability of $2 \times 2850 = 5700$ m/sec.

VII. Planetary Flights

Corresponding to paragraph V, six mission types exist. Only planetary satellites, soft landers and return flights are of interest, as probes are already treated elsewhere.

Perhaps advanced propulsion systems—e.g. ionic systems—will be used for large-scale interplanetary operations, but we will limit ourselves here to the conventional chemical impulsive systems.

Only four representative missions via HOHMANN transfers are considered in some detail, bearing in mind that the return flight has somewhat symmetrical demands.

a) Martian Satellite

As shown, the minimum ideal initial (Earth-side) launch velocity equals 11.5893 km/sec. Mars has to be there, when the vehicle reaches the Aphel of its orbit. The transfer time from Earth is 260 days. Mars moves during these 260 days through $0.524 \cdot 260 = 136$ degrees of arc. Therefore, at launch Mars must be 44 degrees ahead of Earth (which is $\frac{44}{0.987 - 0.524} = 96$ days before opposition). —The Martian orbit is inclined by 1°51′ to the ecliptic. If the vehicle shall go into the Martian plane of motion at the node, the ideal velocity requirement is about $27 \cdot \sin 1°51' = 0.872$ km/sec. By giving the plane change kick about halfway and arriving at Mars moving not within the Martian plane, usually (if the node does

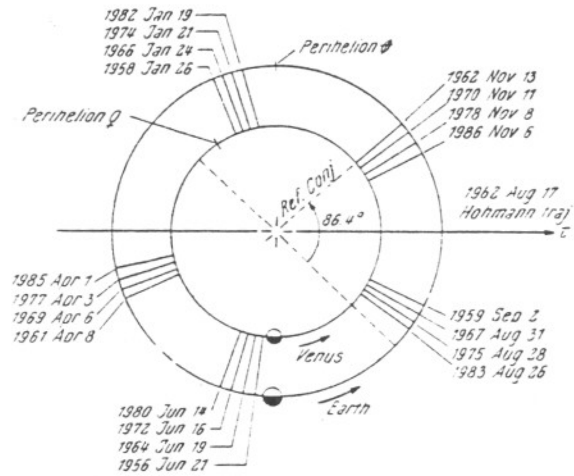

Fig. 12. Inferior conjunction of Venus and HOHMANN ellipse injection date

Hohmann ellipse injection dates:

1956 Mar 26	1967 Jun 4	1978 Aug 13
1957 Oct 31	1969 Jan 8	1980 Mar 18
1959 Jun 6	1970 Aug 15	1981 Oct 23
1961 Jan 10	1972 Mar 21	1983 May 30
1962 Aug 17	1973 Oct 26	1985 Jan 3
1964 Mar 23	1975 Jun 2	1986 Aug 10
1965 Oct 28	1977 Jan 6	— –

Notes: 1. Estimated error of a conjunction or a HOHMANN injection date is 4 days maximum.
2. Orbits and circles; $r(\oplus) = 1.00$ au, $r(♀) = 0.723$ au.
3. Dates and longitudes of inferior conjunctions are: $t =$ JD 243 5062.3/583.931 N (days); and $\lambda = 575°$ 5180 N; Reference date = 1954 Nov 15.29 UT.
4. Constant synodic period = 583.921 (days).
5. HOHMANN date and longitude; $t = 87.6$ days, $\lambda = 86°.4$

not happen to be at this place) some savings can be accomplished. If the vehicle travels neither in the ecliptic nor in the Martian plane, and if the central angle between launch and arrival is slightly smaller than 180 degrees, then the loss due to inclination change becomes negligible. (Oral communication from Dr. D. F. LAWDEN.)

Arrival at the Martian orbit occurs with a velocity of 2.55 km/sec less than Mars orbital velocity. At 1000 km above the Martian surface, the vehicle will move 5.15 km/sec; an ideal minimum braking of 2.01 km/sec brings this down to 3.14 km/sec, which is local circular velocity (escape velocity at zero altitude: 5.04 km/sec).

b) Venusian Satellite

At launch, Venus must be 54.5 degrees behind Earth (or 88 days before inferior conjunction). After 146 days, the vehicle approaches Venus, being 2.5 km/sec faster than Venus. As the inclination of the Venusian orbit is about 3°24′, the maximum requirement of the ideal velocity for the change is $32.5 \sin 3°24' = 1.93$ km/sec.

Escape velocity at Venusian surface is 10.23 km/sec; at 1000 km altitude, this is reduced to 9.49 km/sec (circular velocity: 6.71 km/sec). The vehicle arrives with $\sqrt{(2.5)^2 + (9.49)^2} = 9.814$ km/sec. Therefore, a braking of 3.104 km/sec is necessary.

c) and d) Soft Landings on Mars or Venus

Here I will always assume, that the atmosphere is used for braking and landing. The speed which has to be broken upon arrival is about

Venus: 10.5 km/sec (from Earth)
Earth: 11.5 km/sec (from Mars, Venus)
Mars: 5.64 km/sec (from Earth)

If this is done aerodynamically, then there is only a little fuel used for control. (For the timing and outlay of Martian and Venusian flights, see Figs. 9—12.)

Of some interest might be a manned planetoid mission, because this would give the chance of actually being on another star and doing research there. For a planetoid within Mars' orbit and of negligible gravity, appropriate Mars data can be used.

Appendix

Table I. *Ideal Velocity Requirement for Earth—Bound Missions of Ballistic—Missile Types*

Range (km)	V_{id} (km/sec)
500	3
1,000	3.9
2,000	5
5,000	6.8
10,000	8.5
20,000	8.8

Remarks: According to paragraph III—F, 3% of V_{id} may be added for mixture ratio shifts, trapped residuals and flight performance reserves.

Table II. *Velocity Requirements for Some Missions*
(See Remarks on Table I)

	Values in km/sec	Equatorial Earth Satellites, No Recovery, Equator Launch 96-min	Equatorial Earth Satellites, No Recovery, Launch 24-hr	Equatorial Earth Satellites, No Recovery, AMR 96-min	Equatorial Earth Satellites, No Recovery, Launch 24-hr	Equatorial Earth Satellites, + Recovery, Equator Launch 96-min	Equatorial Earth Satellites, + Recovery, Launch 24-hr	Lunar Missions, No Recovery, Impact	Lunar Missions, No Recovery, Satellite	Lunar Missions, No Recovery, Soft Landing	Lunar Missions, + Return, Circum-Lunar	Lunar Missions, + Return, Satellite	Lunar Missions, + Return, Soft Landing
Earth launch	Ideal Minimum Launch	8.291	11.934	11.801	12.359	8.291	11.934	11.186	11.186	11.186	11.186	11.186	11.186
	Rotational Gain	0.45	0.45	0.4	0.4	0.45	0.45	0.3	0.3	0.3	0.3	0.3	0.3
	G-Loss	1.4	1.42	1.4	1.42	1.4	1.42	1.42	1.42	1.42	1.42	1.42	1.42
	Drag-Loss	0.15	0.16	0.15	0.16	0.15	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	Maneuvering	0.05	0.05	0.01	0.01	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	Maneuvering Transfer	—	0.05	—	0.05	—	0.05	0.05	0.05	0.05	0.05	0.05	0.05
At target arrival	Ideal Minimum	—	—	—	—	—	—	—	1.18	2.89	—	1.18	2.89
	Rotational Gain	—	—	—	—	—	—	—	—	—	—	—	—
	G-Loss	—	—	—	—	—	—	—	0.03	0.3	—	0.03	0.3
	Maneuvering	0.05	0.05	0.05	0.05	0.05	0.05	—	0.05	0.05	0.1	0.05	0.05
Target launch	Ideal Minimum Launch	—	—	—	—	0.06	1.49	—	—	—	—	1.18	2.89
	Rotational Gain	—	—	—	—	—	—	—	—	—	—	—	0.2
	G-Loss	—	—	—	—	—	—	—	—	—	—	0.03	0.15
	Drag-Loss	—	—	—	—	—	—	—	—	—	—	—	—
	Maneuvering	—	—	—	—	—	—	—	—	—	—	0.05	0.05
	Maneuvering Transfer	—	—	—	—	—	—	—	—	—	0.05	0.05	0.05
	Earth Landing Maneuver	—	—	—	—	0.05	0.05	—	—	—	0.05	0.05	0.05
	Total	9.50	13.20	13.10	13.75	9.60	14.75	12.55	13.85	15.80	12.75	15.20	19.35

Earth Landing Maneuver	—	—	—	—	0.05	0.05	—	—	—	0.05	0.05	0.05
Total	9.50	13.20	13.10	13.75	9.60	14.75	12.55	13.85	15.80	12.75	15.20	19.35

Table 11 (Continued)

	Values in km/sec	Space Probes							
		No Recovery							
		Simple Earth Escape	Martian Probe	Venusian Probe	Mercury Probe	HOHMANN Probe + Return	CROCCO Probe + Return	Simple Solar System Escape	Solar Probe
Earth launch	Ideal Minimum Launch	11.2	11.589	11.441	13.504	11.441	13.948	16.658	17.0—31.8
	Rotational Gain	0.4	0.3	0.3	0.3	0.3	0.3	0.4	0.4
	G-Loss	1.42	1.42	1.42	1.45	1.42	1.45	1.46	1.46
	Drag-Loss	0.16	0.16	0.16	0.17	0.16	0.17	0.18	0.18
	Maneuvering	0.05	0.10	0.10	0.10	0.10	0.10	0.10	0.10
	Maneuvering Transfer	—	—	—	—	—	—	—	—
At target arrival	Ideal Minimum	—	—	—	—	—	—	—	—
	Rotational Gain	—	—	—	—	—	—	—	—
	G-Loss	—	—	—	—	—	—	—	—
	Maneuvering	—	0.2	0.2	0.2	0.6 + 1.8 + 2.3	0.6	—	0.2
Target launch	Ideal Minimum Launch	—	—	—	—	—	—	—	—
	Rotational Gain	—	—	—	—	—	—	—	—
	G-Loss	—	—	—	—	—	—	—	—
	Drag-Loss	—	—	—	—	—	—	—	—
	Maneuvering	—	—	—	—	—	—	—	—
	Maneuvering Transfer	—	—	—	—	—	—	—	—
	Earth Landing Maneuver	—	—	—	—	0.1	0.1	—	—
	Total	12.45	13.15	13.0	15.1	17.60	16.05	18.0	18.55—33.35

Table 11 (Continued)

	Values in km/sec	Planetary Missions: No Recovery: Mars Satellite	Venus Satellite	Mars Soft Landing	Venus Soft Landing	+ Return: Mars Satellite	Venus Satellite	Mars Soft Landing	Venus Soft Landing	Planetoid in Mars Orbit, Soft Landing
Earth launch	Ideal Minimum Launch	11.589	11.441	11.589	11.441	11.589	11.441	11.589	11.441	11.589
	Rotational Gain	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	G-Loss	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42
	Drag-Loss	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	Maneuvering	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	Maneuvering Transfer	0.4	0.5	0.4	0.5	0.4	0.5	0.4	0.5	0.4
At target arrival	Ideal Minimum	2.01	3.104	—	—	2.01	3.104	—	—	2.55
	Rotational Gain	—	—	—	—	—	—	—	—	—
	G-Loss	0.1	0.15	—	—	0.1	0.15	—	—	—
	Maneuvering	0.05	0.05	0.01	0.01	0.05	0.05	0.01	0.01	0.05
Target launch	Ideal Minimum Launch	—	—	—	—	2.01	3.104	5.64	10.5	2.55
	Rotational Gain	—	—	—	—	—	—	0.2	—	—
	G-Loss	—	—	—	—	0.05	0.1	0.3	1.5	—
	Drag-Loss	—	—	—	—	—	—	0.15	0.2	—
	Maneuvering	—	—	—	—	0.05	0.05	0.05	0.05	0.05
	Maneuvering Transfer	—	—	—	—	0.04	0.05	0.04	0.05	0.04
	Earth Landing Maneuver	—	—	—	—	0.05	0.05	0.05	0.05	0.05
	Total	15.50	16.60	13.40	13.35	18.05	20.40	19.80	26.15	18.95

Table III. *Velocity Requirements for Various Missions (Equator-Launched, Unless Otherwise Specified)*

On this table the total velocity requirements for a number of missions are summarized. "Direct Target" means, that the maneuver at target is employed without going to an orbit around the target first. "Orbit at Target" on the other hand implies the use of orbital technique at target. The "Elliptical Orbit" is the same as is described in paragraph VI of this report. Also, see remarks on Table I.

	Equatorial Earth Satellites, Circular								Elliptical Equatorial Earth Satellite		Lunar Missions					
	AMR Launched					+ Recovery								+ Return		
	150 km	96 min	24 hr	96 min	24 hr	150 km	96 min	24 hr	150—35871 km	150—35871 km	Impact	Satellite	Soft Landing	Circum-Lunar	Satel-lite	Soft Landing
Direct Target																
Earth	9.20	9.50	13.20	13.10	13.75	—	—	—	11.8	—	12.55	13.85	15.80	—	—	—
Orbit	—	—	3.85	—	4.40	—	—	—	—	—	3.10	4.40	6.30	—	—	—
Earth—Earth	—	—	—	—	—	9.25	9.60	14.75	—	11.9	—	—	—	12.75	15.20	19.35
Orbit—Earth	—	—	—	—	—	—	—	5.4	—	—	—	—	—	3.25	5.75	9.70
Earth—Orbit	—	—	—	—	—	—	—	17.08	—	—	—	—	—	15.90	18.40	22.50
Orbit—Orbit	—	—	—	—	—	—	—	7.73	—	—	—	—	—	6.40	8.85	12.85
Earth—Elliptical Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	13.05	15.45	19.50
Orbit—Elliptical Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	3.55	6.00	10.00
Orbit at Target																
Earth	—	—	—	—	—	—	—	—	—	—	13.7	—	15.95	—	—	—
Orbit	—	—	—	—	—	—	—	—	—	—	4.25	—	6.45	—	—	—
Earth—Earth	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19.45
Orbit—Earth	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9.80
Earth—Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22.60
Orbit—Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12.95
Earth—Elliptical Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19.60
Orbit—Elliptical Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10.10

Table III (Continued)

	Space Probes								Planetary Missions								
							Return						+ Return				
	Earth Escape	Martian	Venusian	Mercury	Solar	Solar System Escape	Hohmann	Crocco	Mars Satellite	Venus Satellite	Mars Soft Landing	Venus Soft Landing	Mars Satellite	Venus Satellite	Mars Soft Landing	Venus Soft Landing	Planetoid Soft Landing
Direct Target																	
Earth	12.45	13.15	13.00	15.1	18.55–33.35	18.0	—	—	15.50	16.60	13.40	13.35	—	—	—	—	15.81
Orbit	3.10	3.70	3.55	5.65	9.20–24.0	8.65	—	—	6.05	7.15	3.95	3.90	—	—	—	—	6.45[1]
Earth—Earth	—	—	—	—	—	—	17.60	16.05	—	—	—	—	18.05	20.40	19.80	26.15	18.95
Orbit—Earth	—	—	—	—	—	—	8.15	6.60	—	—	—	—	8.60	10.95	10.35	16.70	9.50
Earth—Orbit	—	—	—	—	—	—	21.00	21.95	—	—	—	—	21.56	23.76	23.43	20.70	22.50
Orbit—Orbit	—	—	—	—	—	—	11.55	12.50	—	—	—	—	12.11	14.31	13.88	20.25	13.05
Earth—Elliptical Orbit	—	—	—	—	—	—	18.15	19.1	—	—	—	—	18.71	20.91	20.48	26.85	19.65
Orbit—Elliptical Orbit	—	—	—	—	—	—	8.70	9.65	—	—	—	—	9.26	11.46	11.03	17.40	10.20
Orbit at Target																	
Earth	—	—	—	—	—	—	—	—	—	—	15.65	16.75	—	—	—	—	16.00[1]
Orbit	—	—	—	—	—	—	—	—	—	—	6.20	7.30	—	—	—	—	6.55[1]
Earth—Earth	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22.00	29.50	19.10
Orbit—Earth	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12.60	20.10	9.60
Earth—Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	25.55	32.90	22.60
Orbit—Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16.10	23.45	13.15
Earth—Elliptical Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22.70	30.10	19.80
Orbit—Elliptical Orbit	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13.25	20.60	10.30

[1] No return trip included.

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